Draft version August 22, 2025
Typeset using LaTeX **default** style in AASTeX7.0.1

Chi-squared Analysis of Solar Neutrino Parameters at DUNE

Samuel Tello,[1,2] André de Gouvêa,[3] and Olivia Bitter[3]

[1] *Carnegie Mellon University, Department of Physics*
[2] *Northwestern University, Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA)*
[3] *Northwestern University, Department of Physics and Astronomy*

ABSTRACT

Neutrinos are one of the most mysterious and least understood particles in nature, they come in three different flavors that oscillate as they travel through space. Neutrino oscillations were the first conclusive evidence of physics beyond the standard model, and understanding them is highly relevant to some of the big open questions in physics like why the universe is dominated by matter over antimatter. The Deep Underground Neutrino Experiment (DUNE) is optimized for atmospheric neutrino oscillations, but its sensitivity to solar oscillation parameters is less studied. In this project, we tested how sensitive DUNE could be to the solar mixing angle θ_{12} and the mass-squared difference Δm^2_{21}. Using simulated flux data, interaction cross sections, and oscillation probabilities obtained from numerically solving Schrodinger's equation, we created expected event spectra. We then performed a chi-squared analysis to compare expected events for different parameter values. Our results show that DUNE cannot constrain solar parameters as tightly as solar experiments, but it is a good consistency check.

1. INTRODUCTION

Neutrinos are the most abundant particles with mass in the universe, yet they are one of the hardest to detect. They are so elusive because of their extremely small mass and neutral electric charge, which makes it so they hardly interact with anything. Although they are so hard to detect, they have led to the only conclusive evidence of physics beyond the Standard Model, one of the most successful theory in physics. The Standard Model predicts that neutrinos are massless, but the observation of neutrino flavor oscillation directly implies that they have mass. To explain this phenomenon, we must first note that there are three types of neutrinos, each corresponding to a heavier, charged particle (called a lepton) in the standard model: the electron neutrino, the muon neutrino, and the tau neutrino. Oscillation is the process in which a neutrino is created as one of these flavors but is detected as a different one after it has traveled through space. Improving our understanding of these oscillations is helpful in answering fundamental questions in physics, such as what role neutrinos played in the evolution of the early universe and whether could help explain why our universe is dominated by matter over antimatter.

The Deep Underground Neutrino Experiment[1] (DUNE) at Fermilab is an upcoming experiment designed to study neutrino oscillations. DUNE will produce a high intensity neutrino beam and send it from Fermilab to a gigantic detector 1300km away at the Sanford Underground Research Facility in South Dakota. The specifications of DUNE are designed to precisely measure an oscillation parameter linked with atmospheric neutrinos. They will also allow measurement of a parameter called the CP-violating phase, which plays an important role in understanding the dominance of matter over antimatter.

In this project, we focus on testing the sensitivity of the DUNE experiment to a different oscillation parameter associated with neutrinos produced by the sun, rather than atmospheric neutrinos. Solar neutrino oscillation is primarily studied in experiments with low neutrino energies traveling very long distances (approximately one astronomical unit). However, it is important to test the sensitivity of DUNE to this parameter to compare with our existing understanding of neutrino behavior. We do this by computing the theoretical number of neutrinos that would be detected at Sanford Lab for different values of the solar neutrino oscillation parameters. We group the number of detections into different energy intervals or bins, and analyze how the total number of detections varies per bin using a Chi-squared analysis. Our results show that DUNE is not highly sensitive to variations in the solar parameters, but the analysis is a useful consistency check with existing solar neutrino experiments.

2. BACKGROUND

In order to compare the expected number of neutrinos to our chosen value of the solar neutrino parameters, we first have to produce an event spectrum, which is the differential number of events with respect to energy:

$$\frac{dN}{dE} = (\frac{d\Phi_{\nu_\mu}}{dE}\, P_{\mu\to\mu} \,+\, \frac{d\Phi_{\nu_e}}{dE}\, P_{e\to\mu})\, \sigma_\mu \tag{1}$$

Consider some flavor α. For this flavor, the terms in Eq. (1) are as follows: $\frac{d\Phi_{\nu_\alpha}}{dE}$ is the differential neutrino flux, $P_{\alpha\to\mu}$ is the oscillation probability for a neutrino produced as flavor α to be detected as a muon neutrino, and σ_μ is the interaction cross section for a muon neutrino with argon (the target material of DUNE's detector). These are all terms that we can find or compute, and they allow us to calculate the total number events by integrating over the range of neutrino energies:

$$N = \int \frac{dN}{dE}\, dE. \tag{2}$$

In order to get a better physical understanding of each component and where it comes from, we break down each of the terms in Eq. (1).

2.1. *Differential neutrino flux:* $\frac{d\Phi_{\nu_\alpha}}{dE}$

The neutrino flux is the number of neutrinos of a particular flavor α per unit area, per unit time. The differential flux is how this number changes with the neutrino's energy. Given that DUNE will likely not be operational until 2028 , this data comes from simulations . Additionally, in this project we use what is called the "time-integrated" flux, which allows us to hide the unit of time and consider the flux with another unit. Namely, instead of time we consider the number of protons that are used in the process of producing the neutrino beam.

2.2. *Cross section:* σ_μ

In particle physics, when considering the cross section of an interaction, you can think of it as the probability of that interaction occurring. In this case, we are looking at the cross section of a muon neutrino interacting with an Argon atom. Again, this is because the detector at Sanford Labs is going to consist of Argon, and the only flavor of neutrinos it can detect are muon neutrinos (and muon antineutrinos). Similarly to the flux, this value depends on the energy of the neutrino, and the data can be found online.

2.3. *Oscillation probability:* $P_{\alpha\to\mu}$

This is the most nontrivial term to obtain, and one that can be computed by solving the Schrödinger equation for neutrino flavor states. The key idea behind neutrino oscillations is that besides having three flavor states, there are also three neutrino mass states labeled ν_1, ν_2, ν_3, and each of the three flavor states is a linear combination of these three mass states, i.e.,

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}\, |\nu_i\rangle\, . \tag{3}$$

Where the coefficients $U_{\alpha i}$ are elements of a unitary (i.e. $U^\dagger U = I$) matrix called the PMNS matrix:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

where we use the notation $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The indices of these so called *mixing angles* are crucial, because each of the angles, θ_{12}, θ_{13}, and θ_{23}, is half of the picture when we talk about oscillation parameters. For this project, the one that we are interested in is θ_{12}, which you can find in the farthest right matrix in Eq. (4). This is the mixing angle associated with solar neutrino oscillations, and part of the purpose of this project is to see how sensitive DUNE might be to this angle. For context, the angle DUNE is most focused on is θ_{23}, which appears in the leftmost matrix of U and characterizes atmospheric neutrino oscillations.

If you combine Eq. (3) and Eq. (4), we see that this is a change of basis from the neutrino mass basis to the flavor basis. We can multiply together all the matrices that form U and show this explicitly:

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}\\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}\\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}. \tag{5}$$

For example, an electron neutrino written in the mass basis is:

$$|\nu_e\rangle = c_{12}c_{13}\,|\nu_1\rangle \;+\; s_{12}c_{13}\,|\nu_2\rangle \;+\; s_{13}e^{-i\delta}\,|\nu_3\rangle\,. \tag{6}$$

Now we can write the equation we are trying to solve. It looks slightly different from the standard Schrödinger equation because we solve it in terms of distance rather than time. This is valid because we consider the ultra relativistic limit (so $x \approx ct$) and are working with natural units so we set $c = 1$:

$$i\frac{d}{dx}\,|\nu(x)\rangle = H\;|\nu(x)\rangle\,. \tag{7}$$

In the mass basis, the Hamiltonian is relatively simple. It ends up being the masses (corresponding to each mass state, for example, m_1 corresponds to ν_1) squared along the diagonal and a factor of $\frac{1}{2E}$. Formally, $\frac{m_i^2}{2E}$ is the eigenvalue associated with the mass eigenstate ν_i. So this just looks like:

$$H_{\text{mass}} = \frac{1}{2E}\begin{pmatrix} m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix} \tag{8}$$

However, because Hamiltonians are only defined up to an overall phase (i.e. we can add multiples of the identity matrix without changing the physics), we can write this in the more conventional way by subtracting $H_{mass} - \frac{m_1^2}{2E}I$:

$$H_{\text{mass}} = \frac{1}{2E}\begin{pmatrix} 0 & 0 & 0\\ 0 & \Delta m_{21}^2 & 0\\ 0 & 0 & \Delta m_{31}^2\end{pmatrix}, \tag{9}$$

where the Δm_{i1}^2 terms are defined as the difference between the squared masses, so $\Delta m_{i1}^2 = m_i^2 - m_1^2$.

What we are really interested in though is the Hamiltonian in the flavor basis, because we ultimately want to see the time evolution of flavor states. Fortunately, this is not too bad since, if you recall, we already have a matrix to change between flavor and mass bases: the U matrix we saw before (formally called the PMNS matrix).

$$H_{\text{flavor}} = U\,H_{\text{mass}}\,U^\dagger. \tag{10}$$

We still have one more effect that we can not neglect, and it comes from the fact that the neutrinos are traveling through matter (the earth's crust). The electron neutrino has a unique interaction that the other two flavors don't have that makes it so that you have to add an additional term to the Hamiltonian. This term is called the "matter potential". We can fairly approximate it is constant through the earth's crust and just add a constant term to our Hamiltonian:

$$H_{\text{flavor}} = U\,H_{\text{mass}}\,U^\dagger + \begin{pmatrix} A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{11}$$

Now we have the effective Hamiltonian for the flavor basis and we can solve Eq. (7) numerically, and the resulting matrix lets you find the oscillation probability between any two flavors as a function of energy and distance. For our case, we care about $P_{\alpha\to\mu}(E, x)$. And since we know $x = 1300$km for the DUNE experiment, we can find $P_{\alpha\to\mu}(E)$

3. METHODS

3.1. *Differential neutrino flux:*

The initial approach to gather the neutrino flux was to obtain it directly from existing, updated simulation data. However, due to noticeable statistical fluctuation and limited statistics, we chose to recreate an existing plot. We used a digital tool to turn plot data into tabular data. Fig. (1) shows how the recreated data compares to the original plot.

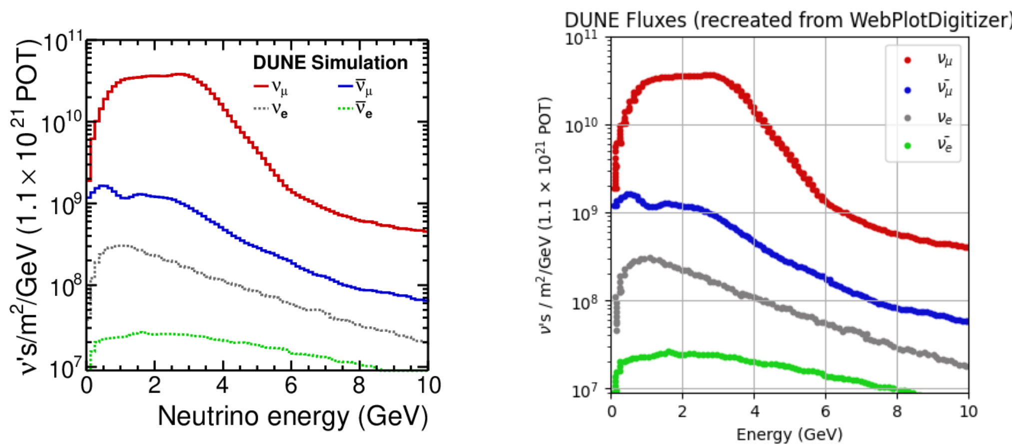

Figure 1. (Left) Official DUNE neutrino fluxes plot (Right) Recreated DUNE fluxes using WebPlotDigitizer

In order to use the data, we first had to create an interpolation. Interpolating made the data easier to use for future calculations. Since we recreated histogram data as a scatter plot, we could not directly interpolate and first had to smooth the data. We do this by applying a Savitzky-Golay filter to smooth the data, then interpolating the smoothed data. as shown in Fig. (2)

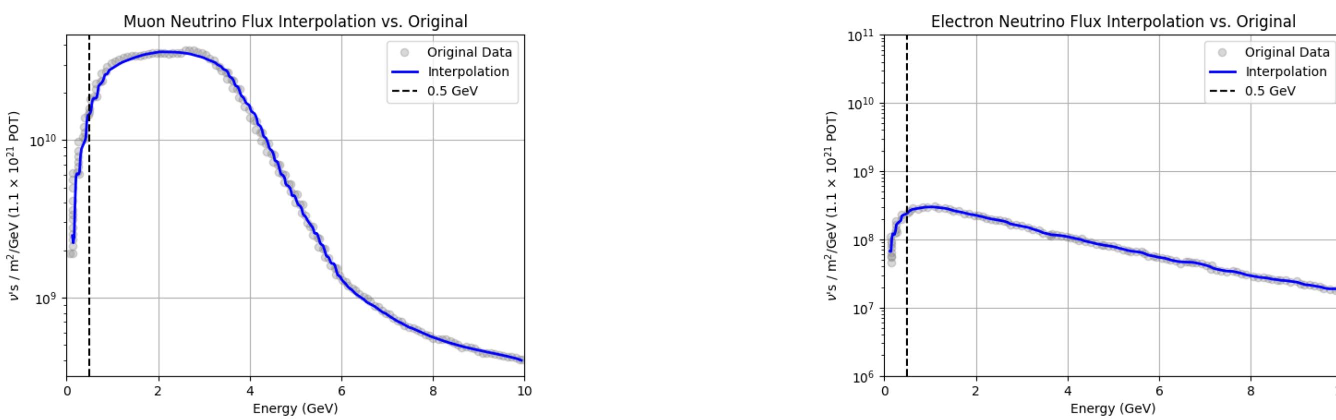

Figure 2. (Left) Muon neutrino flux Akima interpolation after smoothing using a Sav-Gol filter. (Right) Electron neutrino flux interpolation after smoothing by same methods.

3.2. *Cross section:*

The muon neutrino-argon cross section data comes from the General Long Baseline Experiment Simulator[3,4] (GLoBES). The data was a table of cross sections as a function of energy. Similar to the flux, we interpolate our data, but in this case no smoothing was required.

Additionally, we needed to consider that DUNE will deploy a 40 kiloton argon detector. So we multiply our cross section by the total number of argon atoms at the far detector as an overall factor. This corresponds to about $2.4088 \cdot 10^{34}$ argon nuclei.

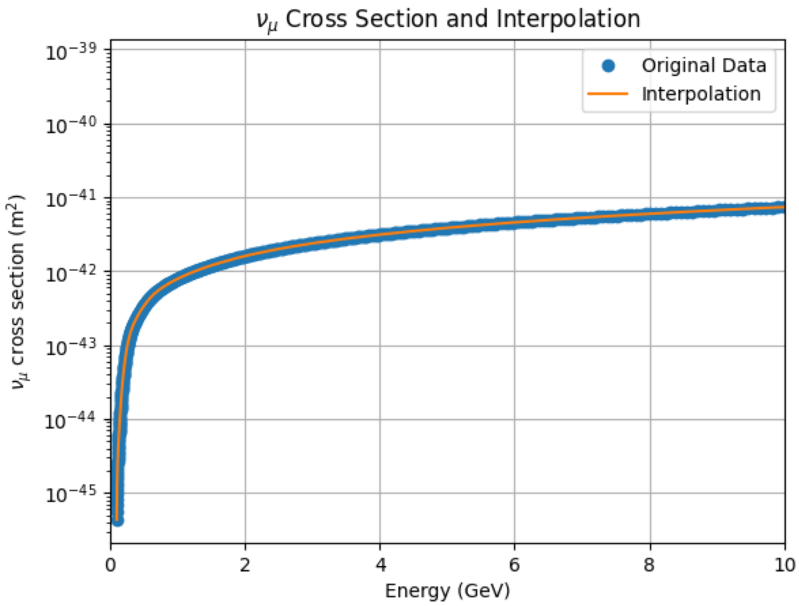

Figure 3. Cross section plot of a muon neutrino on an argon atom from GLoBES data and interpolation.

3.3. *Oscillation probability:*

To obtain the oscillation probabilities, we numerically solved Eq. (7) using solve_ivp from SciPy. The Hamiltonian was created as described in the background section, and the solver ran over 200 iterations between $x = 0$ to $x = 1300$km. Giving us $P_{\alpha\to\mu}(E)$. Fig. (4) shows what the solution looks like. These plots describe the probabilities of a neutrino starting out as one flavor may oscillated into another flavor as it travels 1300km as a function of energy. We note that only neutrinos that were initially electron neutrinos or muon neutrinos are plotted, as these are the only two flavors that can be produced at the beam. We also observe that we are most interested in the orange line in each plot which describes the probability that a neutrino oscillates (or remains) as a muon neutrino. In both plots, we begin at an energy of 0.5 GeV because the oscillation probability formulas begin to break down at low energies.

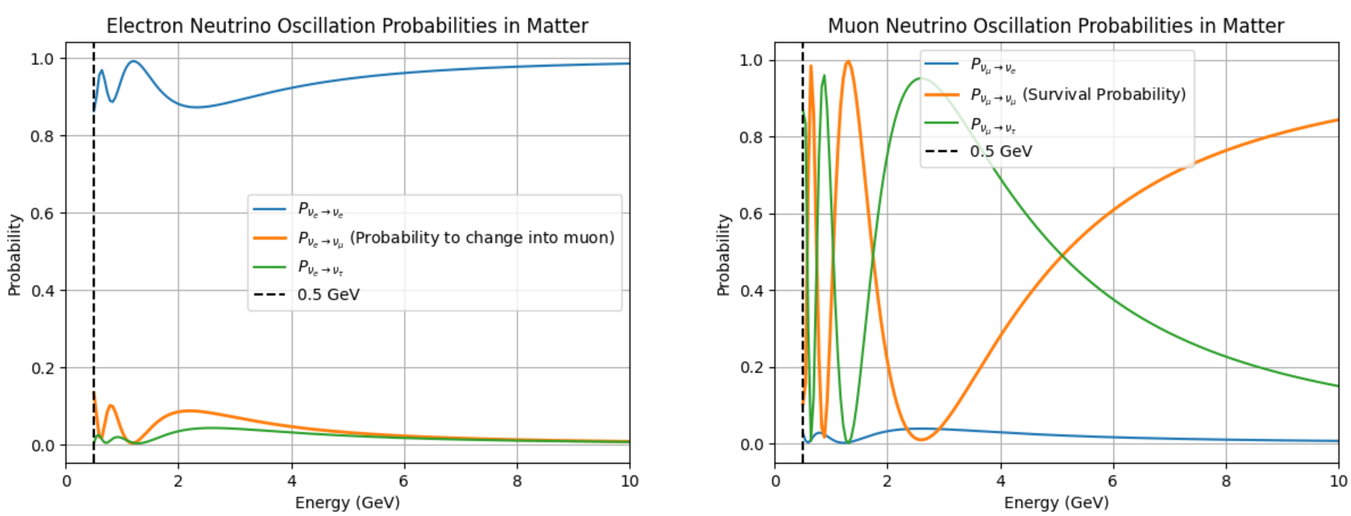

Figure 4. (Left) The probabilities of an (initially) electron neutrino to be detected as a different flavor as a function of energy after traveling from the neutrino beam to the detector. (Right) The same as the left plot, but with a the initial neutrino flavor being a muon neutrino.

3.4. *Event spectrum:*

We multiply all the above components together and create an event spectrum. Shown below in Fig. (5). This is the event spectrum for the currently accepted solar parameters.

The final step before we can iterate over different values of the solar parameters and begin analyzing our data is to turn the differential event spectrum into a total number of events, grouped into bins. This is to reflect the way the detector actually collects data. In Fig. (5) you can see the histogram where we integrated over different energy ranges to get the total number of events per energy range. This histogram is how we will run our chi-squared analysis.

Now, we iterate over different values of θ_{12} and Δm^2_{21}. For each, we create distinct bins to perform a χ^2 analysis. The χ^2 value,

$$\chi^2 = \sum_i \frac{(N_i - E_i)^2}{\sigma_i^2},$$

tells us how well the number of events in each energy bin (N_i) matches the bins created when using the accepted value (E_i). The value σ_i represents the statistical uncertainty in each bin. We use the standard statistical uncertainty, and set $\sigma_i = \sqrt{E_i}$.

In our analysis, we choose the accepted values from NuFit[2] as $\theta_{12}^{(\mathrm{acc})} = 0.588$ (radians) and $\Delta m_{12}^{2\,(\mathrm{acc})} = 7.49 \cdot 10^{-5}\ \mathrm{eV}^2$. To test the dependence of the integrated event spectrum on each parameter individually, We perform the χ^2 test for 20 evenly spaced points in the range $\theta_{12} \in [0, \frac{\pi}{2}]$ and $\Delta m^2_{12} \in [0, 15 \cdot 10^{-5}]$. To test both simultaneously we use the same ranges, and create a 20 x 20 heat map from 400 chi-squared tests.

4. RESULTS

4.1. *Integrated event spectrum for accepted parameters*

Fig. (5) shows what the integrated and binned event spectrum looks like using the accepted values from NuFit. We notice peaks at the [0.5,1.5] GeV bin and [3.5,4.5] GeV bin, which corresponds to the peaks of the muon neutrino survival probability.

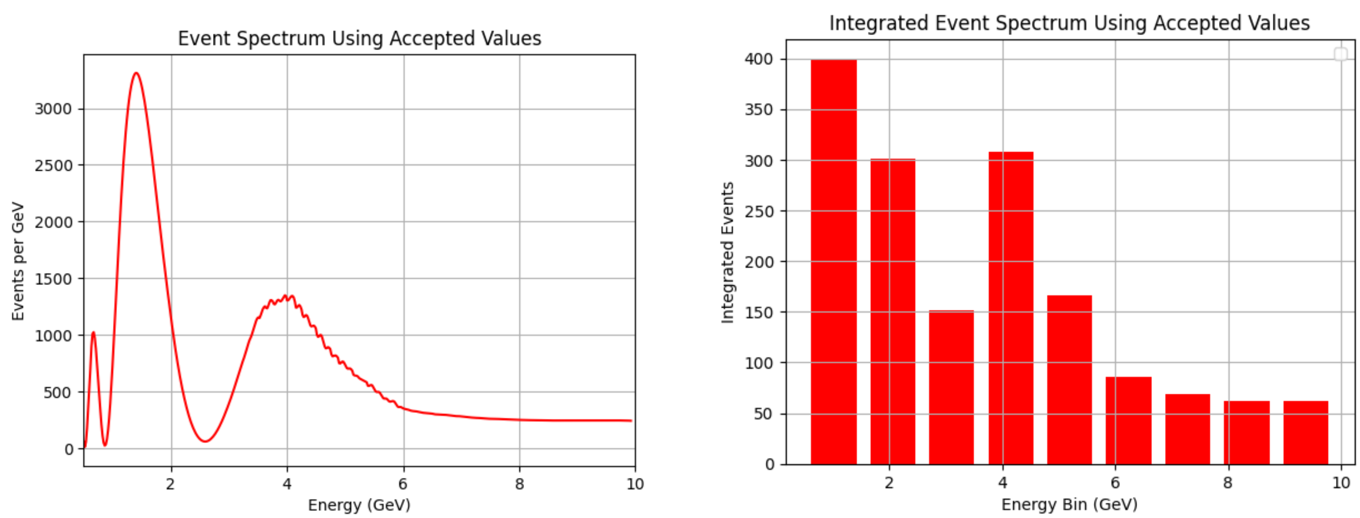

Figure 5. (Left) The differential event spectrum as described by Eq. (1). (Right) The event spectrum integrated over distinct energy bins to more closely recreate how detectors collect data.

4.2. *Varying θ_{12}*

Fig. (6) shows a plot of our chi-squared curve as a function of $\sin^2(\theta_{12})$ to match the observable. We see that DUNE does not constrain the values of θ_{12} well and is particularly weak for small values .

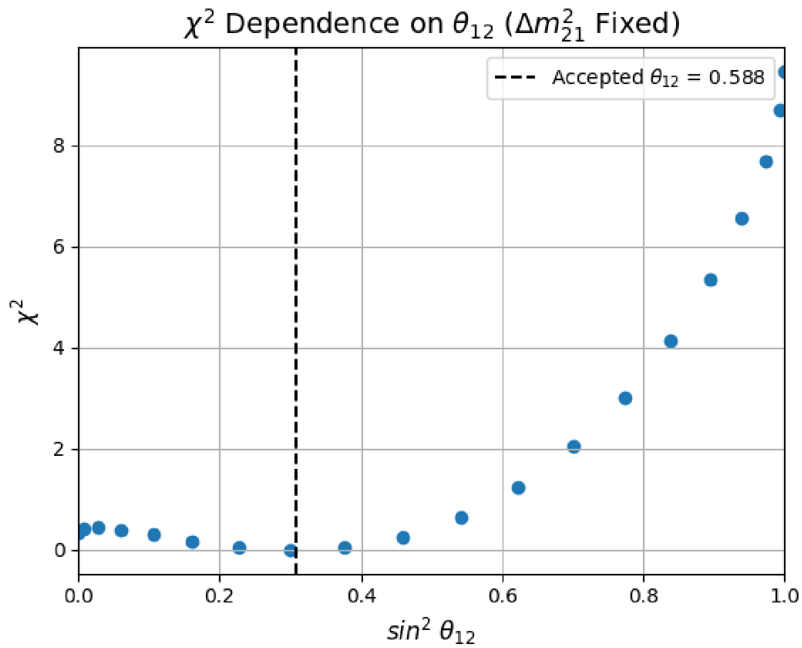

Figure 6. The chi-squared curve created from varying θ_{12} between $[0, \frac{\pi}{2}]$. We use $\sin^2(\theta)$ as this is the actual observable rather than theta itself.

4.3. *Varying* Δm^2_{21}

Fig. (7) shows our plot of the chi-squared curve as a function of Δm^2_{21}. We again find that it is constrained weakly, but it is better constrained than θ_{12}.

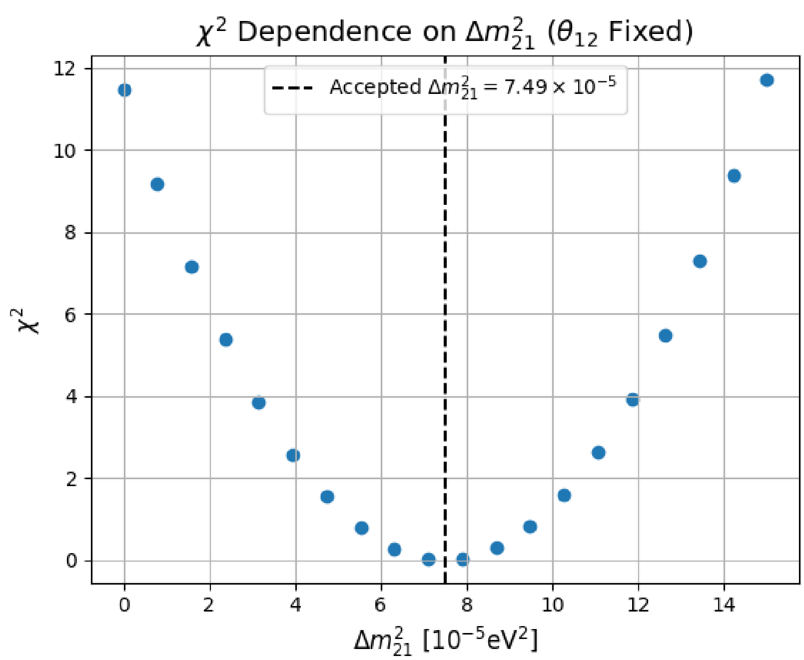

Figure 7. The chi-squared curve created from varying Δm^2_{21} between $[0, 15 \cdot 10^{-5}\ \mathrm{eV}^2]$.

4.4. *two parameter chi-squared test*

Finally, Fig. (8) is a heatmap where we varied both solar oscillation parameters. We can see some of the constrained regions as well as the large regions that DUNE can not constrain, highlighting its limited sensitivity.

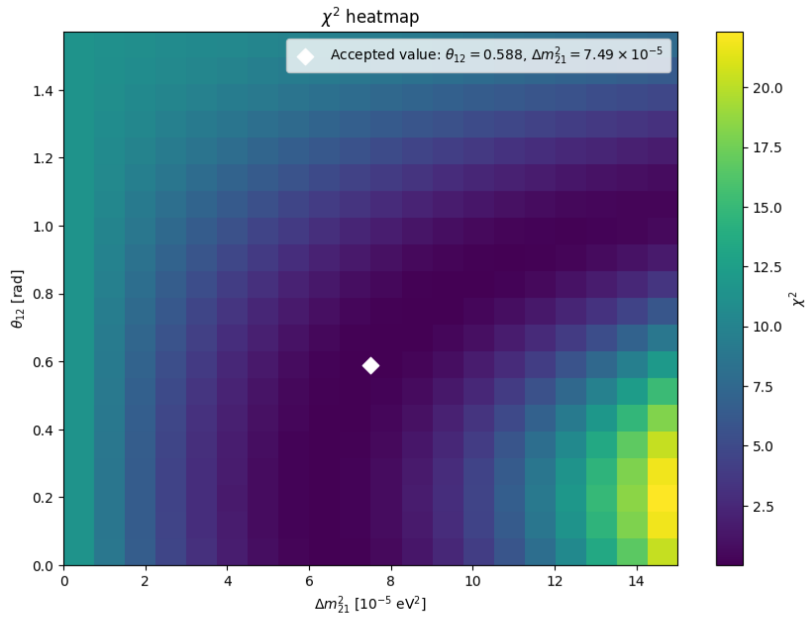

Figure 8. The Chi-squared heat map created by varying both θ_{12} and Δm^2_{21}, making a 20 x 20 grid with 400 iterations.

5. DISCUSSION

Our analysis shows that DUNE has little sensitivity to the solar oscillation parameters. This is consistent with expectations, as DUNE is optimized for atmospheric oscillations. The chi-squared tests show that the variations of the parameters do not lead to strong variations from our accepted data.

There are several limitations in the analysis. First, we assumed perfect energy resolution, meaning the detector can distinguish perfectly between any arbitrarily close energies. This is not realistic and will be accounted for in future work. Similarly, our analysis does not account for detector noise and backgrounds, which would change the shapes of the event spectra. Including these effects would give a more realistic picture of DUNE's sensitivity.

Another point of interest is the χ^2 dependence on θ_{12}. Although the graph is consistent with our expected low sensitivity, the shape of the graph will require further analysis. Future work will clarify if this behavior is expected.

6. CONCLUSION

In this project, we tested DUNE's sensitivity to variations of the solar neutrino oscillation parameters. We created an event spectrum, calculated the predicted number of events across different energy bins, and used these data to perform a chi-square analysis.

We found that DUNE has limited sensitivity to solar parameters relative to solar experiments. However, DUNE can still test these parameters and give a valuable consistency check under different conditions than those of solar experiments.

Future work would involve including more detector effects and understanding the chi-squared dependence on θ_{12}. This will give us a more realistic picture of DUNE's sentitivity to solar neutrino oscillations.

REFERENCES

[1] DUNE Collaboration, Far Detector Technical Design Report, Vol. I: Introduction to DUNE, JINST 15 (2020) T08008. doi:10.1088/1748-0221/15/08/T08008.

[2] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, A. Zhou, NuFIT 6.3 (2024), www.nu-fit.org.

[3] P. Huber, M. Lindner, W. Winter, Simulation of long-baseline neutrino oscillation experiments with GLoBES, Comput. Phys. Commun. 167 (2005) 195. doi:10.1016/j.cpc.2005.01.003.

[4] P. Huber, J. Kopp, M. Lindner, M. Rolinec, W. Winter, New features in the simulation of neutrino oscillation experiments with GLoBES 3.0, Comput. Phys. Commun. 177 (2007) 432. doi:10.1016/j.cpc.2007.05.004.